Workshop in Game Theory

Friday

9:15	Welcome
9:30-10:20	Sharing sequential profits in a network Rubén Juárez
10:20-11:10	Extensions and additional results for cost sharing problems with non rival consumption William Olvera López
11:10-12:00	Linear symmetric rankings Luís Hernández Lamoneda 12:00-12:30
Coffee break	
12:30-13:30	Fair Division using Topological Combinatorics Francis Edward Su
	Lunch
15:00-15:30	A Solution for a Multiple Cost Sharing Problem Jony Rojas Rojas
15:30-16:00	A new solution for Games in Generalized Characteristic Function Form Oliver Antonio Juárez Romero
16:00-16:50	A variation of the top trading cycle Francisco Sánchez Sánchez
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Workshop in Game Theory

Saturday

9:00-9:50	Two possible explanations for centipede game paradox Leobardo Plata Pérez
9:50-10:40	Dynamic Assignment of Objects to Queuing Agents with Private Values David Cantala
10:40-11:30	Tucker's Lemma: an application in consensus-halving and a new proof based on a volume argument Mutiara Sondjaja
11:30-12:00	Coffee break
12:00-12:50	Voting in Agreeable Societies Francis Edward Su
	Lunch
14:30-15:00	Games with constraints given by a nested structure of cooperation Miguel Vargas Valencia
15:00-15:30	Two solutions for transferable utility games based on the Talmudic recommendation for the disputed garment problem Iván Téllez Téllez
15:30-16:30	Ham Sandwich Theorems Ted Hill
18:30-	Dinner

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Sharing sequential profits in a network

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Abstract

Consider a process where agents bring individual values which might be different at every possible step. A planner is in charge of choosing steps and distributing the aggregate value of the process. In particular, a problem consists of an acyclic-directed-network with a common source and multiple sinks. There is a finite number of agents. Each link in the network contains a vector with non-negative numbers which represents the marginal contribution of the agents if this link is selected. A path connecting the source and a sink has to be chosen and the value of the path must be distributed among the agents.

In the problem with complete information, the planner knows the marginal contribution of the agents at every link in the network and is interested in systematically selecting a path and share the profits meeting four axioms: *continuity* in the vector of contributions, *non-triviality* in the selection of non-zero paths, *value-monotonicity* in the profit of agents relative to increase in marginal contributions, and *composition* in the way to select paths, where agents are indifferent between receiving a lump-sum payment at the end of the process or receiving installment at any point in time. The combination of these four axioms characterizes a family of path selection rules with maximal value and profit-sharing in fixed proportions that are independent of the network. Two alternative axiomatizations of related rules are also provided.

Alternatively, in the problem with incomplete information, the planner might be uninformed. He knows the structure of the network but might not know the marginal contributions of the agents. We focus in the case where the planner might be able to delegate the decision to more informed agents, and thus he is interested in finding a sharing rule that incentivize the agents to select the efficient path as a subgame perfect Nash equilibrium of a game. We show that for a large class of games, most of the rules obtained under incomplete information coincide with the rules found in the case of complete information.

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Extensions and additional results for cost sharing problems with non rival consumption

Julio Macias-Ponce * William Olvera-Lopez †

Abstract

In this work we show several results related to the discrete cost sharing problem with non rival consumption: First, we show alternative characterizations and additional properties for a well-known solution based on prices for this problem, which implies to calculate a Shapley value for allocating prices to the services (or goods) and then, to divide this price, by each service, among the agents in some equalitarian way. Second, we show that this solution is close related to the Owen value of a transferable utility cooperative game (TU-game) where the associated coalition structure considers each single unit of service required as an agent. Under this line, we provide some ideas about using coalitional structures for finding solutions to the problem. On addition, we propose and alternative solution based on the classic axioms that characterize the Shapley value for TU-games and, removing one of the axioms that we use in our characterization, we show a family of solutions for the problem, as well as some properties that every member of the family of solution satisfies. After that, we change the original model considering that each agent who requires a service could require it in different quantities (or to different number of persons): this implies to consider a modification of the original cost function, and we provide a characterization of a solution in this situation.

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Linear symmetric rankings

Luís Hernández Lamoneda

Abstract. Every power index induces a hierarchy on the players involved by looking at the way their payoffs are ordered. This gives rise to a well defined notion of ranking as an equivalence relation on the set of all power indices.

In this note we do a, more or less, systematic study of all rankings arising from linear and symmetric indices. This study leads to a classification of all "positive" (in a certain precise way) rankings: they are in 1-1 correspondence with the points of the standard simplex. This correspondence has certain nice geometrical properties (eg it is convex and maps Shapley to the barycenter). Finally, this parametrization simplex is contrasted with the one proposed by Saari-Sieberg, using positive semi-values.

This is joint work with Francisco Sánchez.

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Voting in Agreeable Societies

Francis Edward Su

Abstract. When does a majority exist? How does the geometry of the political spectrum influence the outcome? We will show how some classical mathematics about the geometry of convex sets and their intersections can be used to model people's preferences and understand voting in "agreeable" societies.

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A Solution for a Multiple Cost Sharing Problem

Jony Rojas Rojas

Abstract. We work with a cost sharing problem with non-rival consumption where the cost of the services depends on the agents who require them. We propose a solution based on cooperative games for finding a price for each service according to the cost function and the demand of each agent and also for finding how to distribute this price, by service, among the agents who require it. We show a characterizations of the proposed solution.

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A new solution for Games in Generalized Characteristic Function Form

Oliver Antonio Juárez Romero

Abstract. Generalized characteristic functions extend characteristic functions of 'classical' TU-games by assigning a real number to every ordered coalition being a permutation of any subset of the player set. Such generalized characteristic functions can be applied when the earnings or costs of cooperation among a set of players depend on the order in which the players enter a coalition.

In the literature, the two main solutions for generalized characteristic functions are the one of Nowak and Radzik (1994), shortly called NR-value, and the one introduced by Sánchez and Bergantiños (1997), shortly called SB-value.

In this work we introduce a new solution for this kind of games. This solution is based on the generalization of the potential of Hart and Mas-Colell (1989) for classical TU-games. We also provide an axiomatization of this solution.

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A variation of the top trading cycle

Francisco Sánchez Sánchez

Abstract. Let there be n traders in a market, where each agent manages only one good (whose amount is a known datum) and where we further assume that all goods are different from each other. To each agent there's assigned a linear utility function.

The aim of this paper is to model a bartering market in such a way that all agents increase their utilities by the same amount. We say that the barter is even if it has this property. In this work, we find the expression for all even barters, in particular, for the optimal one.

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Two possible explanations for centipede game paradox

Leobardo Pedro Plata Pérez

Abstract. The centipede game paradox is generated by backward induction. Two rationality weakened are presented in order to explain the paradox. First introduces a discount rate to model players impatience. In the second one uncertainty is introduced to see the play as a Markov chain.

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Dynamic Assignment of Objects to Queuing Agents with Private Values *

Francis Bloch† David Cantala‡ July 13, 2015

Abstract

This paper analyzes the optimal assignment of objects which ar- rive sequentially to agents organized in a waiting list. Applications in- clude the assignment of social housing and organs for transplants. We analyze the optimal design of probabilistic queuing disciplines, pun- ishment schemes, the optimal timing of applications and information releases. We consider three efficiency criteria: the vector of values of agents in the queue, the probability of misallocation and the expected waste. Under private values, we show that the first-come first-served mechanism dominates a lottery according to the first two criteria but that lottery dominates first-come first-serve according to the last cri- terion. Punishment schemes accelerate turnover in the queue at the expense of agents currently in the waiting list, application schemes with commitment dominate sequential offers and information release always increases the value of agents at the top of the waiting list.

Mutiara Sondjaja
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Title:
Tucker's Lemma: an application in consensus-halving and a new proof based on a volume argument.

Abstract:
Tucker's lemma is a combinatorial theorem about centrally-symmetric triangulations of the n-dimensional sphere. It states that if the vertices of such triangulations receive a label from $\{\pm 1, \pm 2, \ldots, \pm n\}$ such that the labels of antipodal vertices sum to zero, then there must exists a pair of adjacent vertices whose labels sum to zero. Tucker's lemma is known to be equivalent to the Borsuk-Ulam theorem, a topological results which has applications in fair division problems.

We present a result of Su and Simmons (2002) in which Tucker's lemma is used to construct a solution to the consensus-halving problem: how to divide an object into two portions so that each of n people believes that the two portions are equal. Then, we present a new proof of Tucker's lemma that is based on a volume argument.

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Fair Division using Topological Combinatorics

Francis Edward Su

ABSTRACT. The Brouwer fixed point theorem is a beautiful and well-known theorem of topology that has a combinatorial analogue known as Sperner's lemma. There are also several other combinatorial analogues of topological theorems. In this talk, I will explain applications to problems of fair division, and trace recent connections and generalizations.

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Games with constraints given by a nested structure of cooperation

Miguel Vargas Valencia *
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January 5, 2016

1 Abstract

There are real situations where a set of agents have mechanisms to reach agreements. Usually those situations are modeled as transferable utility cooperative games (TU games). One of the main problems is the distribution of payoffs between the agents. When *a priori* coalitional agreements exist, such situations are modeled using cooperative games with coalitional structures (that is, a partition of the set of players). An extension of the Shapley value for these games was presented by Owen (1977). However, sometimes the information of the cooperation structure provided by the coalitional structure is insufficient. Specifically, the coalitional structure may not describe the situations where cooperation arises on several levels (with multiple partitions of the set of players). For this case, Winter (1989) introduced games with a *level structure* and presented an extension of *the Owen value* for this structure.

In the classical model of cooperative games, any set of players is a feasible coalition. Indeed, a cooperative game may be modeled as a real-valued function defined on the power set of players. But in some situations, cooperation between players can be restricted by exogenous conditions which prevent the formation of some of these coalitions. Some examples of restricted cooperation are presented in Myerson (1977) and Owen (1986), for communication games; Faigle and Kern (1992), for cooperative games under precedence constraints; Bilbao and Edelman (2000), for games on convex geometries; and Koshevoy and Talman (2014), for games with general coalitional structure. In these papers, the Shapley value is adapted to propose solutions that conform to the respective models.

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Here we present games with a coalitional structure similar to the one presented by Winter (1989), but with an important conceptual difference. The cooperation among players is restricted by constraints given in the nested structure, thus the domain of the *characteristic function* is not the power set of players. In this structure, the set of players is classified into several levels according of a family of partitions of the set of players, $(\mathcal{P}^0, \mathcal{P}^1, ..., \mathcal{P}^{p+1})$, where each partition is a refinement of the previous one. Throughout the paper, $\mathcal{P}^0$ denotes the partition of singletons and the last partition $\mathcal{P}^{p+1}$ stands for the partition of the grand coalition, these will be called trivial partitions. The elements of each partition will be called *classes*. Every class has a set of subclasses, this is the set of elements in the previous partition contained in the class. In fact, subclasses provide a partition of the class to which they belong. In our model, only subsets of players are considered *feasible coalitions* if they satisfy nested classification constraints in the following sense: a coalition S is the union of subclasses belonging to the same class. The games defined on the set of feasible coalitions will be called *cooperative games with nested structures* or simply *nested games*. The classical situation occurs exactly when the set of players only has the trivial partitions. In this way, we consider the nested structure as *a rule to form* a coalition and we consider that only *feasible coalitions* can have a value. Furthermore, we give solutions for this new type of games defined in this model. Specifically, we develop adaptations of the Shapley value and of the weighted Shapley value for these games since the proposed values for level structure model use the profit of some likely unfeasible coalitions, Gómez-Rúa and Vidal-Puga (2011); Winter (1989). In the particular case that there is only one partition other than the trivial, $(\mathcal{P}^0, \mathcal{P}^1, \mathcal{P}^2)$, our adaptations of the Shapley value matches the *Collective value* presented by Kamijo (2013).

In this talk we introduce our model by describing the set of feasible coalitions. We show some algebraic properties of this set as a partially order set with the relation $\subseteq$. Those properties are the principal tool for giving a representation of a nested game with respect to a basis of unanimity nested games. We propose and characterize axiomatically a Shapley value for nested games by first defining it on the basis of unanimity nested games and extending it linearly. To interpret the adaptation of the Shapley value, we will define a game for each class where its players will be the subclasses that comprise it. With the weighted Shapley value of these games, where the weights are the cardinalities of the subclasses, we define a distribution among the agents that make up such subclass.

Moreover, we suggest a multi-stage process that provides an algorithm to calculate the proposed values. With this process, a simple and intuitive vision is obtained for the allocation rule developed throughout the talk. Finally, in accordance with the work of Owen in (Roth, 1988, sec. 10), we define a *multilinear extension* for the nested games. This extension allows us to compute in a different way the adapted Shapley value.

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Two solutions for transferable utility games based on the Talmudic recomendation for the disputed garment problem

Iván Téllez Téllez *

Abstract

In this work we use the Talmudic solution, for the disputed garment problem, to propose two solutions for transferable utility games. This solution, seen as a "concede and divide" rule, gives a way to assign the disputed parts of a good. First, we use this two person solution to share $v(N)$ between S and $N \setminus S$ and divide equally the results among the members of each coalition. We study the expected value, of the payments that a player receives, over all coalitions he joins. The result obtained is a linear, symmetric and efficient solution and can be written as a convex combination of the Shapley value and the egalitarian solution. We give an example where this solution assigns "almost" equally the value of the grand coalition if the number of players is big enough.

For the second solution we generalize the concede and divide rule to solve n-person problems as follows. Using the concede and divide rule, the nonempty coalitions S and $N \setminus S$, divide $v(N)$. Now that every coalition considered has a state, we can iterate this process for both coalitions until we reach singlets with a payment for each one. Since the way of doing this is not unique, we first count the number of paths in which the grand coalition can be successively splitted in nested partitions until we get only singlets. We propose, as solution for our game, the average of payments over all paths and compute a closed expression for it. Finally, as our proposed solution is related to bankruptcy problems, we give examples to compare solutions to some bankruptcy problems and our solution to games related to these bankruptcy problems.

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Speaker: Ted Hill
Title: "Ham Sandwich Theorems"

The classical Ham Sandwich Theorem of Steinhaus says that given any three bounded objects (Borel sets) in Euclidean three-space, there is always a plane that simultaneously bisects (the volume of) all three objects, for example, bread, ham, and cheese. This talk will briefly discuss the following: history of the theorem, its relationship to game theory and fair division extensions and generalizations (to other dimensions, other cutting surfaces, and other measures), related partitioning results including the centerpoint theorem, several applications, and open problems. The talk is aimed for the non-specialist.